Name:

Fall 2017 Math 245 Exam 2

Please read the following directions:
Please write legibly, with plenty of white space. Please print your name on the designated line, similarly to your quizzes (last name(s) in ALL CAPS). Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will last at most 50 minutes; pace yourself accordingly. Please leave **only** at one of the designated times: 1:20pm, 1:40pm, or 1:50pm. At all other times please stay in your seat (emergencies excepted), to ensure a quiet test environment for others. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
Exam Total:	50		100
Quiz Ave:	50		100
Overall:	50		100

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

a. predicate

b. $\forall x \in D, P(x)$

c. counterexample

d. Proof by Contradiction Theorem

Problem 2. Carefully define the following terms:

a. Nonconstructive Existence theorem

b. Proof by Induction

c. Proof by Reindexed Induction

d. Proof by Strong Induction

Problem 3. Prove that for all $n \in \mathbb{N}$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Problem 4. Prove or disprove: $\forall x \in \mathbb{Z},\ |7x + 20| > 1$.

Problem 5. Prove or disprove: $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R},\ x^2 < y^2 < x^2 + 1$.

Problem 6. Prove or disprove: $\exists y \in \mathbb{R}\ \forall x \in \mathbb{R},\ x^2 < y^2 < x^2 + 1$.

Problem 7. Let F_n denote the Fibonacci numbers. Prove that $\forall n \in \mathbb{N}$, $F_{2n} = \sum_{i=0}^{n-1} F_{2i+1}$.

Problem 8. Let $x \in \mathbb{R}$. Prove that $2\lfloor x \rfloor \leq \lfloor 2x \rfloor \leq 2\lfloor x \rfloor + 1$.

Problem 9. Let $n \in \mathbb{N}$. Prove that there is at most one $a \in \mathbb{N}$ satisfying $a^2 \leq n < (a+1)^2$.

Problem 10. Prove that $\sqrt{5}$ is irrational.